

# Application of the MACBETH approach to coalition formation

MARC ROUBENS<sup>1,2</sup>, AGNIESZKA RUSINOWSKA<sup>3,4</sup>, and  
HARRIE DE SWART<sup>3</sup>

<sup>1</sup> University of Liège, Institute of Mathematics, B37, B-4000 Liège

<sup>2</sup> Faculté Polytechnique de Mons, Rue de Houdain 9, B7000 Mons, Belgium  
m.roubens@ulg.ac.be

<sup>3</sup> Tilburg University, Department of Philosophy, P.O. Box 90153  
5000 LE Tilburg, The Netherlands  
a.rusinowska@uvt.nl, H.C.M.deSwart@uvt.nl

<sup>4</sup> Warsaw School of Economics, Department of Mathematical Economics  
Al. Niepodleglosci 162, 02-554 Warsaw, Poland  
arusin@sgh.waw.pl

## EXTENDED ABSTRACT

Based on semantic judgements concerning the attractiveness of available alternatives, MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique, see [1], [2], and [www.m-macbeth.com](http://www.m-macbeth.com)) is an interactive approach to quantify the attractiveness to a given party or agent of each alternative, in such a way that the measurement scale constructed is an interval scale. In this paper, we present an application of the MACBETH approach ([1], [2]) to a model of coalition formation ([3]). The main concept of the model presented in [3] is the concept of a stable government, where a government is defined as a pair consisting of a majority coalition and a policy supported by this coalition.

We apply the MACBETH technique to quantify the attractiveness and repulsiveness of possible governments to parties. We use this method to calculate the utilities (the values) of governments to parties.

Let  $N$  be the set of all parties. There are  $M \geq 1$  independent policy issues on which a government will have to decide. Let  $P$  and  $P_j$  denote a policy space and a policy sub-space on issue  $j$ , respectively. A policy is represented by a tuple  $p = (p_1, \dots, p_M) \in P$ , where each  $p_j \in P_j$  is a policy on issue  $j$ . A majority coalition will be denoted by  $p_{M+1}$ , and the set of all majority coalitions will be denoted by  $P_{M+1}$ . A *government* is a pair  $g = (p, p_{M+1})$  consisting of a majority coalition  $p_{M+1} \in P_{M+1}$  and a policy  $p = (p_1, \dots, p_M) \in P$  proposed by this coalition. A *stable government* is defined as a government which is not dominated by any other feasible government (see [3]).

In the present model, each party is assumed to have preferences regarding all governments with respect to certain criteria. The criteria are the policy issues and an ‘extra’ issue: ‘majority coalition’. Let  $C^*$  be the set of all criteria, that is,  $C^* = \{1, \dots, M, M+1\}$ , where criterion  $M+1$  concerns the ‘majority coalition’. For each  $j \in C^*$ , each party orders all policies on issue  $j$  taking into account the attractiveness of these policies on the given issue. For each issue  $j$ , each party is

asked to specify two particular references: *neutral*, defined as ‘neither satisfying nor unsatisfying’, and *good*, which is more attractive to a party than *neutral*, and is defined as ‘undoubtedly satisfying’. A *neutral policy on issue  $j$  for party  $i$*  is denoted by  $p_j^{i,0}$ , and a *good policy on issue  $j$  for party  $i$*  is denoted by  $p_j^{i,*}$ .

For each  $j \in C^*$  and  $i \in N$ , based on references  $p_j^{i,0}$  and  $p_j^{i,*}$ , we distinguish:

- an *unattractive (or repulsive) policy on issue  $j$  to party  $i$*  if it is less attractive to  $i$  than  $p_j^{i,0}$
- an *attractive (or simply attractive) policy on issue  $j$  to party  $i$*  if it is more attractive to  $i$  than  $p_j^{i,0}$
- a *very attractive (or outstanding) policy on issue  $j$  to party  $i$*  if it is at least as attractive to  $i$  than  $p_j^{i,*}$ .

For each criterion  $j \in C^*$ , each party  $i \in N$  is asked to verbally judge the difference of attractiveness between each two policies on issue  $j$ ,  $p_j$  and  $p'_j$ , where  $p_j$  is at least as attractive to  $i$  as  $p'_j$ . When judging, a party has to choose one of the following categories:

- $C_0$  - *no* difference of attractiveness
- $C_1$  - *very weak* difference of attractiveness
- $C_2$  - *weak* difference of attractiveness
- $C_3$  - *moderate* difference of attractiveness
- $C_4$  - *strong* difference of attractiveness
- $C_5$  - *very strong* difference of attractiveness
- $C_6$  - *extreme* difference of attractiveness.

For each criterion  $j \in C^*$ , each party  $i \in N$  orders all policies on issue  $j$  from the best one, denoted by  $p_j^{i,1}$ , to the worst one, denoted by  $p_j^{i,|P_j|}$ . Let  $U_j^i(p_j)$  denote the value of policy  $p_j \in P_j$  on issue  $j \in C^*$  to party  $i \in N$ . Let  $K = \{1, 2, 3, 4, 5, 6\}$ . We apply MACBETH to find the values of the policies on issues which satisfy the following rules (see, for instance, [1]):

$$\forall i \in N \forall j \in C^* \forall p_j, p'_j \in P_j :$$

$$U_j^i(p_j) > U_j^i(p'_j) \Leftrightarrow p_j \text{ is more attractive to } i \text{ than } p'_j \quad (1)$$

$$\forall i \in N \forall j \in C^* \forall k, k' \in K \forall p_j, p'_j, p''_j, p'''_j \in P_j \text{ with } (p_j, p'_j) \in C_k \text{ and}$$

$$(p''_j, p'''_j) \in C_{k'} : k \geq k' + 1 \Rightarrow U_j^i(p_j) - U_j^i(p'_j) > U_j^i(p''_j) - U_j^i(p'''_j) \quad (2)$$

If rules (1) and (2) are satisfied, which means that the matrix of judgements is consistent, then the software determines, from all the possible scales, a particular scale (called the MACBETH basic scale) by a procedure which consists essentially of solving a certain linear programme. For each  $i \in N$  and  $j \in C^*$ , the objective function in this programme is  $\min U_j^i(p_j^{i,1})$  under the following constraints:

$$U_j^i(p_j^{i,|P_j|}) = 0 \quad (3)$$

$$\forall p_j, p'_j \in P_j \text{ with } (p_j, p'_j) \in C_0 : U_j^i(p_j) = U_j^i(p'_j) \quad (4)$$

$$\forall k, k' \in K \cup \{0\} \text{ with } k > k', \forall (p_j, p'_j) \in C_k \forall (p''_j, p'''_j) \in C_{k'} :$$

$$U_j^i(p_j) - U_j^i(p'_j) \geq U_j^i(p''_j) - U_j^i(p'''_j) + k - k' \quad (5)$$

If it is impossible to satisfy rules (1) and (2), then no interval scale can represent the judgements expressed. The matrix of judgements is then not consistent, and a message appears on the screen ('inconsistent judgements') inviting the party to revise its judgements. The MACBETH software warns the party and shows the way(s) to obtain a cardinally consistent matrix of judgements. When having the MACBETH basic scale calculated, we can also transform it (into the MACBETH transformed scale), assigning value 0 to a neutral policy  $p_j^{i,0}$  on issue  $j \in C^*$  and, for instance, value 100 to a good policy  $p_j^{i,*}$  on issue  $j$ , for party  $i \in N$ . The basic MACBETH scale and each transformed MACBETH scale are still a *pre-cardinal scale*. In order to obtain a *cardinal scale*, a discussion with the party in question around the scale takes place. When a party thinks that finally the scale adequately represents the relative magnitude of the judgements, we have the *cardinal scale* and the (final) values of all policies on the given issue.

Let  $V_j^i(p_j)$  denote the (final) value of policy  $p_j \in P_j$  on issue  $j \in C^*$  to party  $i \in N$ . We assume then

$$\forall i \in N \forall j \in C^* : V_j^i(p_j^{i,*}) = 100 \text{ and } V_j^i(p_j^{i,0}) = 0. \quad (6)$$

Moreover, we get negative values for all repulsive policies on issue  $j \in C^*$  to party  $i$ , and values greater than 100 for all outstanding policies on issue  $j$  to party  $i$ . Finally, we want to measure the global attractiveness of each government, that is, the attractiveness of each government taking all criteria into account. We adopt the following aggregation procedure:

$$V^i(g) = V^i(p_1, \dots, p_M, p_{M+1}) = \sum_{j=1}^{M+1} \alpha_j^i \cdot V_j^i(p_j), \quad (7)$$

$$\text{with } \alpha_1^i, \dots, \alpha_{M+1}^i \geq 0, \text{ and } \sum_{j=1}^{M+1} \alpha_j^i = 1. \quad (8)$$

In a similar way as in the case of policies on a given issue, we may apply the MACBETH software to calculate the scaling constants  $(\alpha_j^i)_{j \in C^*}$ , where  $\alpha_j^i$  is called the *weight of criterion  $j \in C^*$  to party  $i \in N$* . Finally, using the aggregation procedure given in (7), one may calculate the values of all governments to each party and identify stable governments if there are any.

## References

- [1] C.A. Bana e Costa, J.C. Vansnick, The MACBETH approach: basic ideas, software and an application, in: N. Meskens, M. Roubens (Eds.), *Advances in Decision Analysis*, Kluwer Academic Publishers, Dordrecht, 1999, pp. 131-157.
- [2] C.A. Bana e Costa, J.M. De Corte, J.C. Vansnick, MACBETH, LSE OR Working Paper 03.56, 2003.
- [3] A. Rusinowska, H. de Swart, J.W. van der Rijt, A new model of coalition formation, forthcoming in *Social Choice and Welfare*, 2004.